CSCI 6214— Randomized Algorithms— FALL 2021
Problem Set #1

Problem 1:

Suppose we flip a coin n times to obtain a sequence of flips $X_1, \cdots, X_n$. A *streak* of flips is a consecutive sequence of flips that are all the same. For example if X_3, X_4, X_5 are all heads then there is a streak of length 3 starting at the third flip (if X_6 is also heads, then there is also a streak of length 4).

- Let n be a power of 2. Show that the expected number of streaks of length $\lfloor \log_2 n + 1 \rfloor$ is $1 + o(1)$.
- Show that for sufficiently large n, the probability that there is no streak of length at least $\log_2 n - 2\log_2 \log_2 n$ is at most $\frac{1}{n}$. (hint: break the sequence of flips into disjoint blocks of $\lfloor \log_2 n + 1 \rfloor$ flips).

Problem 2:

Let $a_1, \cdots, a_n$ be a list of n distinct numbers. We say that a_i and a_j are inverted, if $i < j$ but $a_i > a_j$. The Bubblesort sotring algorithm swaps pairwise adjacent inverted numbers in the list, until there are no more inversions, so the list is in sorted order. Suppose that the input to Bubblesort is a random permutation, equally likely to be any of the $n!$ pemutations of n distinct numbers. Determine the expected number of inversions that need to be corrected by the algorithm.

Problem 3:

Consider a simplified version of the roulette, in which you wager x dollars to either red or black. The wheel is spun, and you receive your wager plus another x dollars if the ball lands on your color. If the ball doesn't land on your color, you lose your wager. Each color occurs independently with prbability 1/2. The following strategy is a popular one: on the first spin, bet a dollar. If you loose, bet two dollars on the next spin. In general, if you have lost on the first $k-1$ spins, bet 2^{k-1} dollars on the k-th spin. Argue that following this strategy, you will eventually win a dollar. Now let X be a random variable that measures your maximum loss before winning (i.e. the amount of money you have lost before the play on which you win). Show that $E(X)$ is unbounded. What does this imply about the practicality of your strategy?