CSCI 6214— Randomized Algorithms— FALL 2021
Problem Set #6

Problem 1:
*(*** BONUS PROBLEM*** requires basic understanding of SDP duality, which you can obtain by reading relevant material on the web.)*

Given a graph $G(V, E)$ the dual of the max-cut SDP is:

minimize $\frac{1}{2}m + \sum_{i=1}^{n} \gamma_i$
subject to $A + \text{diag}(\gamma)$ is positive semidefinite.

Here, A is the adjacency matrix of the graph, and $\text{diag}(\gamma)$ is a diagonal martrix with the γ_i in the diagonal. Show that the value of every feasible solution for the dual is an upper bound on the weight of the maximum cut. Repeat the question, if the graph is weighted where each edge ij has a positive weight w_{ij}.

Problem 2:

For the unweighted triangle graph (three vertices, three edges), the max- cut value is 2. Show that the maximum value of the SDP relaxation is exactly $\frac{9}{4}$. Use the primal SDP to give a lower bound and the dual SDP to give an upper bound. If you did not do question 1, then just use it as a given.

Problem 3:

Consider the following variation of the max-cut problem that we refer to as max restricted cut. The (weighted) graph $G(V, E)$ has $2n$ vertices, and the vertices are arranged in pairs: for every $1 \leq i \leq n$, vertex i is paired with vertex $n + i$. Edges have positive weights w_{ij} . A legal cut is one in which for every pair, the two vertices in the pair are in different sides of the cut. The objective is to find a legal cut that maximizes the total weight of edges that are cut. Present an SDP relaxation for max restricted cut.