CSCI 7000-005 Computational Complexity
Problem Set 1

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due September 13, 2018

Collaboration Policy: The homework can be worked on in groups of up to 3 students each (2 would be optimal, but 1 and 3 are both accepted).

One submission per team is sufficient. Please write the solution for each of the problems on a separate sheet of paper. Write your team's names and id on each submission and please **staple** all the sheets together.

Submissions should be written in LaTeX, unless your handwriting is indistinguishable from LaTeX.

Homework is due before the end of class, September 13. Only one late homework per person will be allowed. If you submit more than one homework late, you will get no grade for the excess late homeworks.

Problem 1 (25 pts.)

Consider the following decision problem, that we call U: we are given in input (M, x, t, l) where M is a Turing machine, $x \in \{0,1\}^*$ is a possible input, and t and l are integers encoded in unary, and the problem is to determine whether there is a $y \in \{0,1\}^*, \|y\| \leq l$, such that $M(x, y)$ accepts in at most t steps. Show that U is NP-complete.

Problem 2 (25 pts.)

Define a language L which belongs to SIZE($O(1)$) and is undecidable.

Problem 3 (25 pts.)

Recall that NEXP is defined by

$$NEXP = \cup_c NTIME(2^{n^c}), c \geq 1$$

Give a definition of NEXP that does not involve non-deterministic Turing machines, analogous to the verifier definition of NP seen in class, and prove that your definition is equivalent to the above definition using non-deterministic Turing machines.

Problem 4 (25 pts.)

Prove that if $P = NP$, then $EXP = NEXP$.