Computational Complexity. Lecture 10

NL=coNL

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Today

- Randomized log-space
- Alternate characterization of NL
- NL=coNL

Randomized log-space

- Introduce randomized space-bounded TM (for simplicity only for decision problems).
  - Read-only input tape
  - Read/write work tape
  - Read-only random tape with one-way access (the head can only move from left to right)
- For every fixed input and fixed content of random tape, TM is completely deterministic and either accepts or rejects.

Randomized log-space

- For machine M, input x, random tape content r, denote M(r,x) the outcome of the computation.
- Decision problem L belongs to the class RL if there is a probabilistic TM M that uses O(log n) space on inputs of length n and such that
  - For every $x \in L, Pr_r[M(r,x)\ \ accepts] \geq \frac{1}{2}$
  - For every $x \notin L, Pr_r[M(r,x)\ \ accepts] = 0$

Randomized log-space

- Any constant bigger than zero and smaller than one would work.
- Follows that L⊆RL⊆NL

- Even though we now know that L=SL, it is interesting to see the "old" proof of SL⊆RL.
- **Theorem**. The problem ST-UCONN is in RL.

An alternate characterization of NL

- We saw alternate definition of NP that used certificates instead of non-determinism.
- Can we do the same for NL?
- Certificates might be poly length.
- Need to assume that they are provided to a log-space machine on a read only tape.

An alternate characterization of NL

- **Definition.** A language L is in NL if there exists a deterministic TM M (verifier) with an additional read-once tape, and a polynomial p:$\mathbb{N} \rightarrow \mathbb{N}$ such that for every $x \in \{0,1\}^*$
  $x \in L \Leftrightarrow \exists u \in \{0,1\}^{p(|x|)}$ s.t. M(x,u)=1

- By M(x,u) we denote the output of M where x is placed on the input tape and u on the special read-once tape, and M uses only O(log(|x|)) space on its work tapes for every input x.

An alternate characterization of NL

- What if we remove the read-once restriction and allow the TM's to move back and forth on the certificate?
- This changes the class from NL to NP (ex).

NL=coNL

- Analogously to coNP, we define coNL to be the class of languages that are complements of NL languages.
- Complement of STCONN is in coNL, denote it by $\overline{STCONN}$: Given directed graph G and special vertices s,t decide whether t is NOT reachable from s.
- In fact, it is coNL-complete.

NL=coNL

- Will show that there is an NL TM which solves $\overline{STCONN}$.
- Generally, for every “well behaved” s(n), NSPACE(s(n))=coNSPACE(s(n)). (ex)