CSCI 5444: Introduction to Theory of Computation

Lecture 01: Introduction

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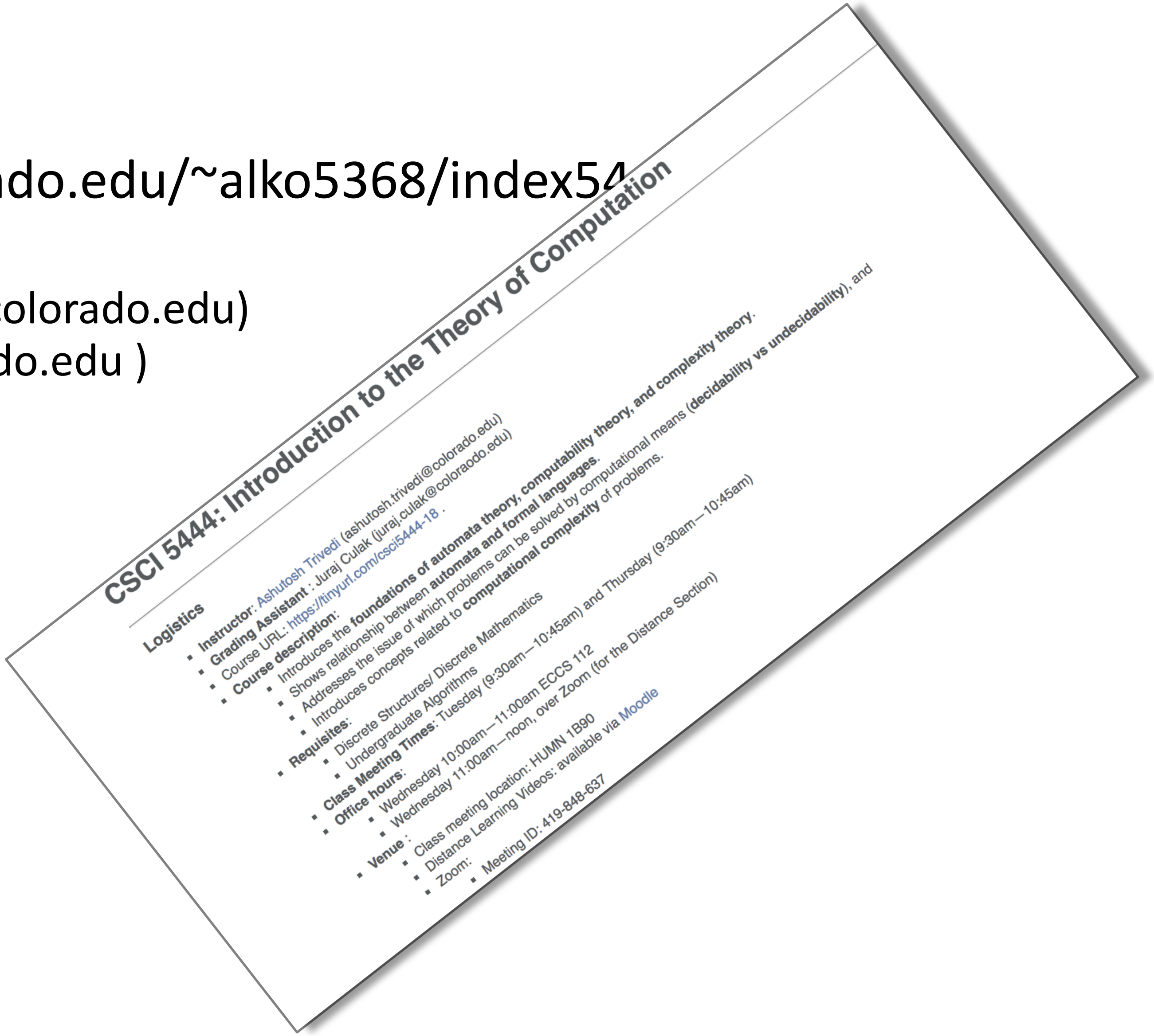
Logistics

- Web-page: http://home.cs.colorado.edu/~alko5368/index54
- Instructor and Grading Assistant
  - Alexandra Kolla (Alexandra.Kolla@colorado.edu)
  - Charles Carlson (chca0914@colorado.edu)
- Lectures
  - Tuesday (11:30am – 12:15am)
  - Thursday (11:30am – 12:15am)
- Office hours
  - TBD
  - By appointment
- Venue
  - Class: HUMN 1B90
  - Office hours: ECCS 122



CSCI 5444: Introduction to the Theory of Computation

Logistics

- **Instructor**: Ashutosh Trivedi (ashutosh.trivedi@colorado.edu)
- **Grading Assistant** : Juraj Culak (juraj.culak@coloraodo.edu)
- Course URL: https://tinyurl.com/csci5444-18 .
- **Course description**:
  - Introduces the **foundations of automata theory, computability theory, and complexity theory**.
  - Shows relationship between **automata and formal languages**.
  - Addresses the issue of which problems can be solved by computational means (**decidability vs undecidability**), and
  - Introduces concepts related to **computational complexity** of problems.
- **Requisites**:
  - Discrete Structures/ Discrete Mathematics
  - Undergraduate Algorithms
- **Class Meeting Times**: Tuesday (9:30am—10:45am) and Thursday (9:30am—10:45am)
- **Office hours**:
  - Wednesday 10:00am—11:00am ECCS 112
  - Wednesday 11:00am—noon, over Zoom (for the Distance Section)
- **Venue** :
  - Class meeting location: HUMN 1B90
  - Distance Learning Videos: available via Moodle
  - Zoom:
    - Meeting ID: 419-848-637

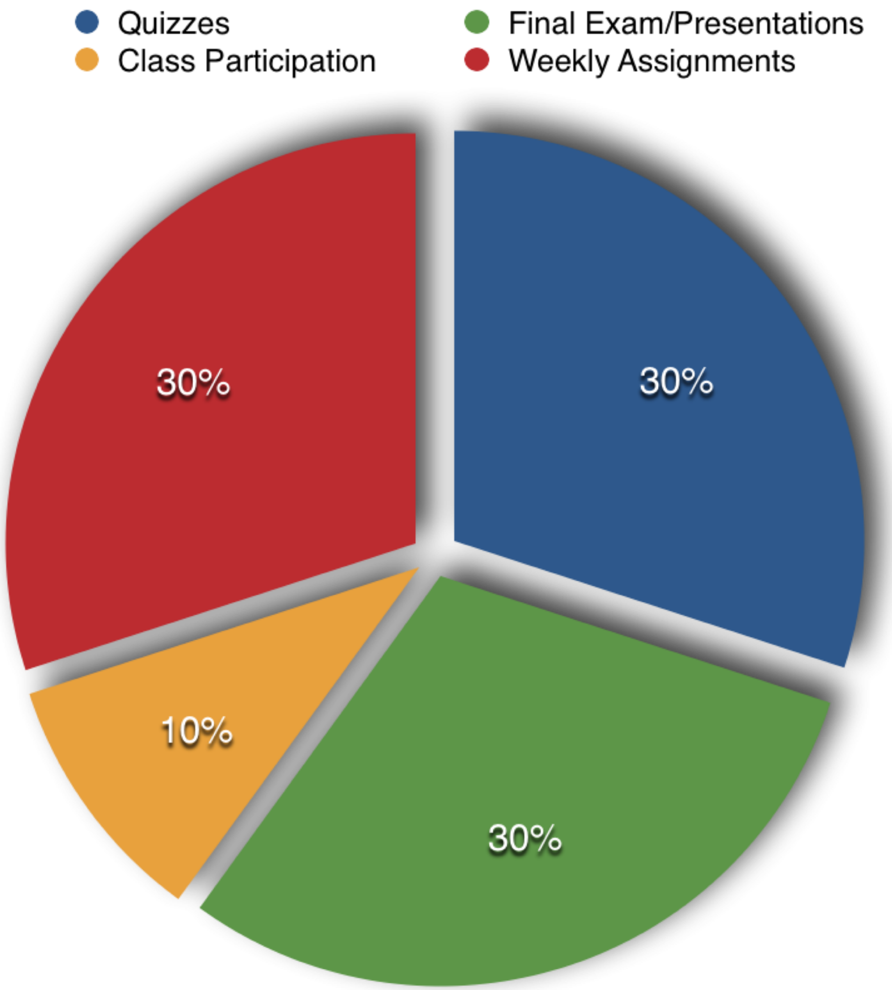
Logistics (Contd.)

- Requisite
  - Discrete Structures (CSCI 2824) / Discrete Mathematics
  - UG Algorithms (CSCI 3104)
- Textbook
  - *Michael Sipser*. Introduction to the Theory of Computation, PWS Publishing Company.
- Other supplemental materials
  - Automata and Computability , Dexter C. Kozen
  - Automata Theory, Languages, and Computation , Hopcroft, Motwani, and Ullman (3rd edition).
  - Elements of the theory of computation , Lewis and Papadimitriou (2nd edition).
  - Descriptive Complexity, Neil Immerman
  - Elements of Finite Model Theory, Leonid Libkin
  - Computational Complexity, Sanjeev Arora and Boaz Barak
  - Online notes and readings distributed by instructor

Logistics (Contd.)

- Zoom
- Moodle
  - All assignments will be posted on moodle.
  - Your identikey is needed for signing in.

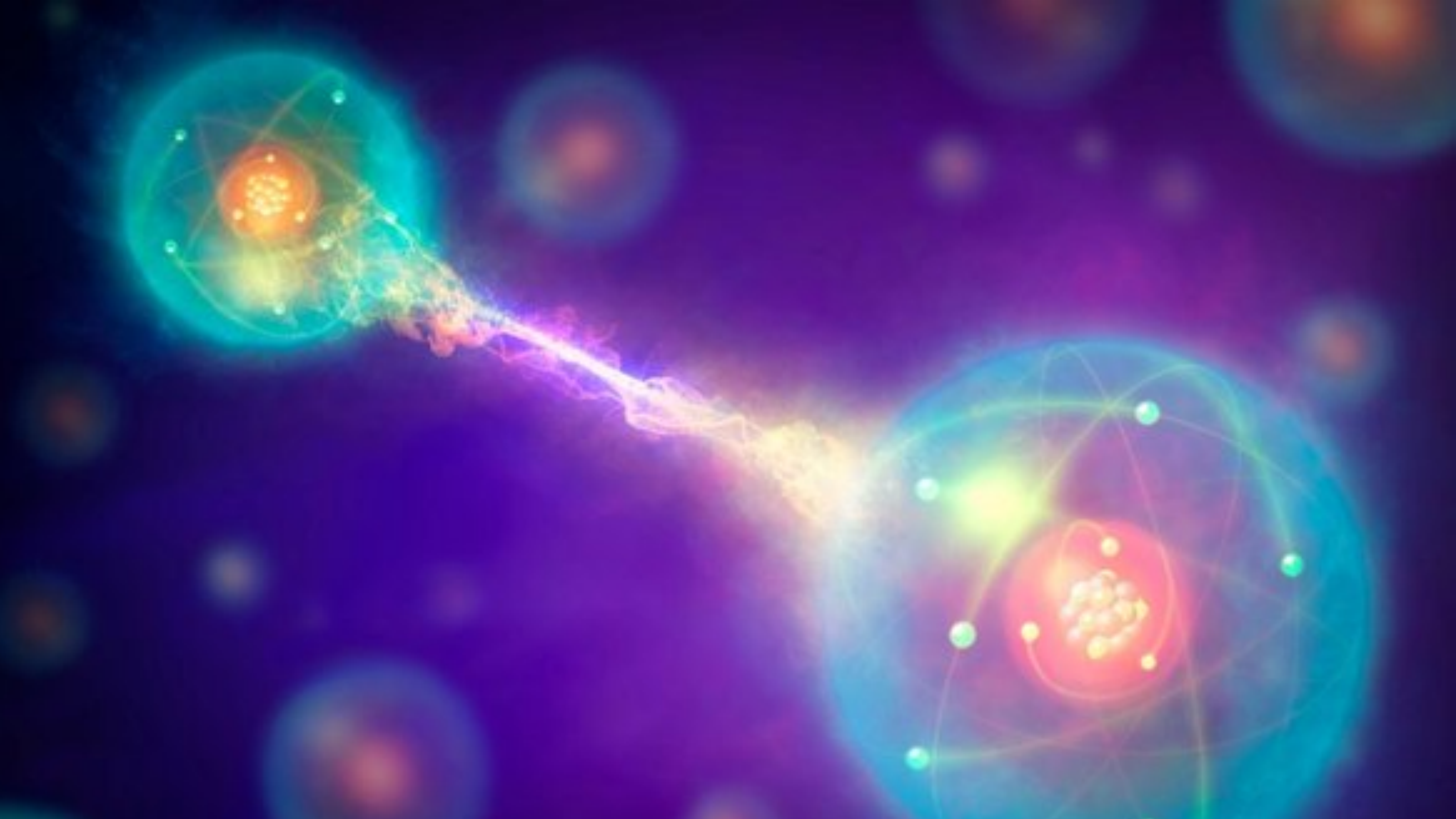
Logistics: Grading

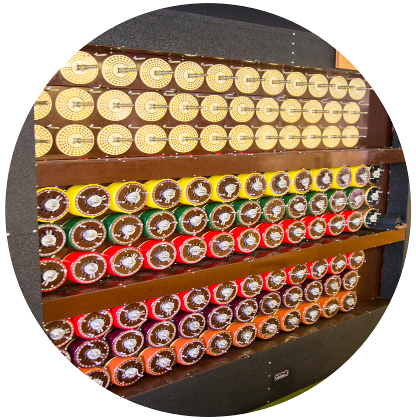
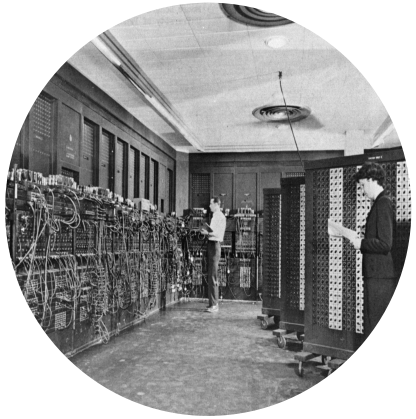


Find this week's assignment on The websit (moddle to come shortly)

Theory of Computation

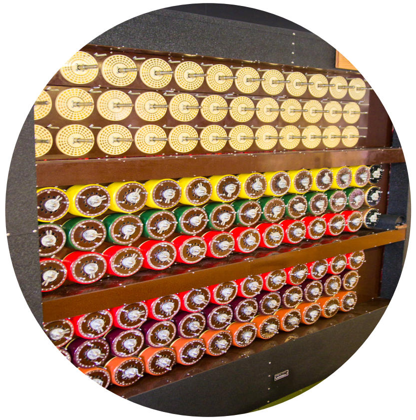
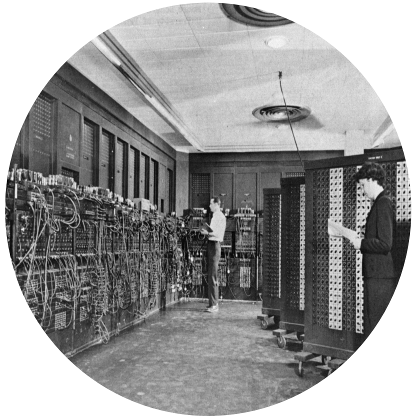
What are the fundamental capabilities and limitations of computers?



What are the fundamental capabilities and limitations of computation?

- What do we mean by computation?
- What is a problem?
- Are all problems computable?
- What is an “efficient” computation?
- Are some problems inherently more difficult than others?

What are the fundamental capabilities and limitations of computers?

- How do we model “computational machines”?
- Are all computational machines **equally powerful**?
- Why should we study computationally weaker models?
- Why a practically-oriented computer-programmer should learn theory of computation?

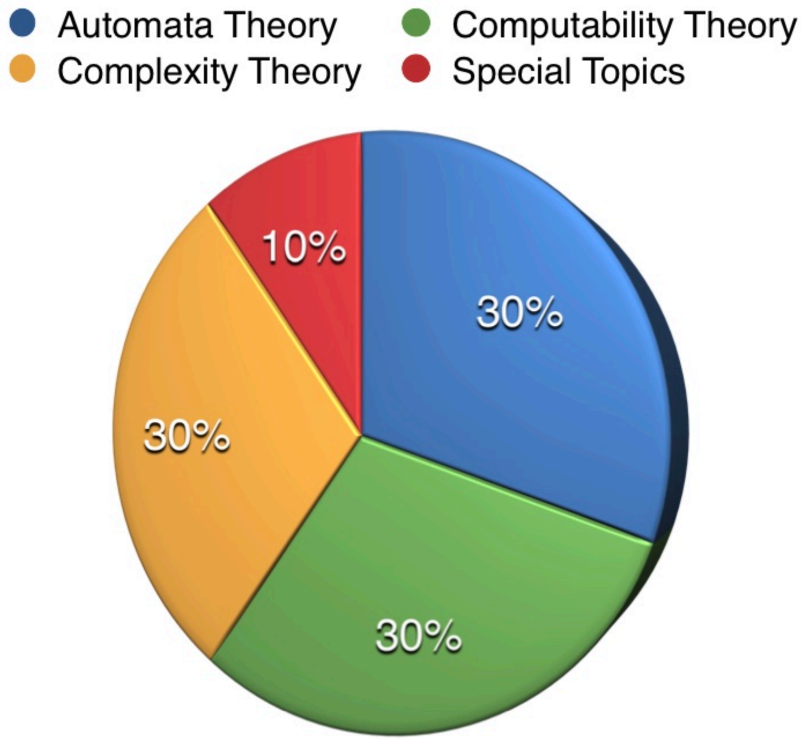
Theory of Computation

Automata Theory

- Formalization of the notion of problems via **formal languages**
- Formalization of the notion of computation using "abstract computing devices" called **automata**
- Understanding a hierarchy of classes of problems or formal languages (regular, context-free, context-sensitive, decidable, and undecidable)
- Understanding a hierarchy of classes of automata (finite automata, pushdown automata, and **Turing machines**)
- Understanding applications to pattern matching, parsing, and programming languages

Computability Theory

Complexity Theory



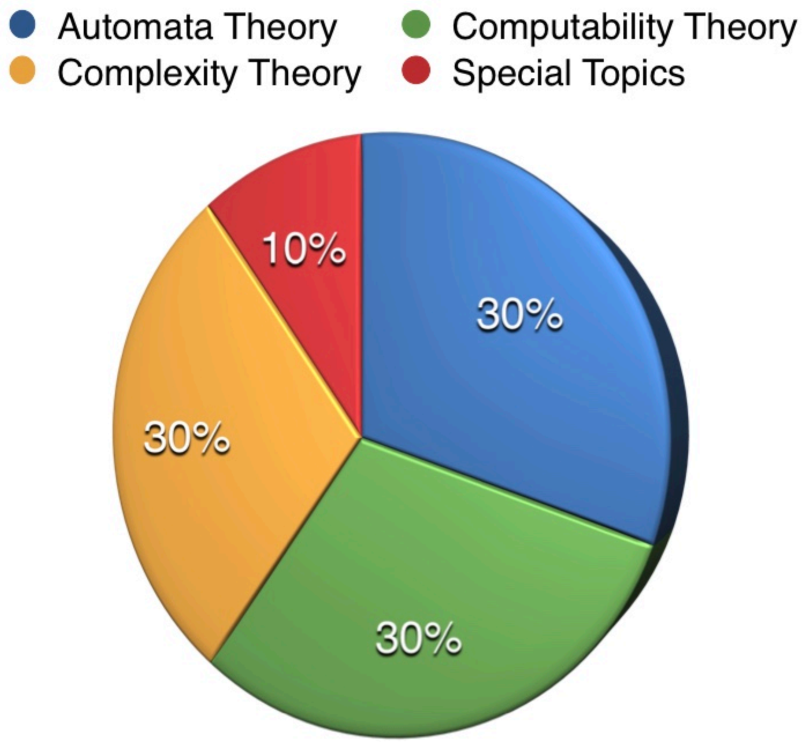
Theory of Computation

Automata Theory

Computability Theory

- Understanding **Church-Turing thesis** (Turing machines as a notion of "general-purpose computers")
- Understanding the concept of **reduction** , i.e., solving a problem using a solution (abstract device) for a different problem
- Understanding the concept of **undecidability** , i.e., when a problem can not be solved using computers

Complexity Theory



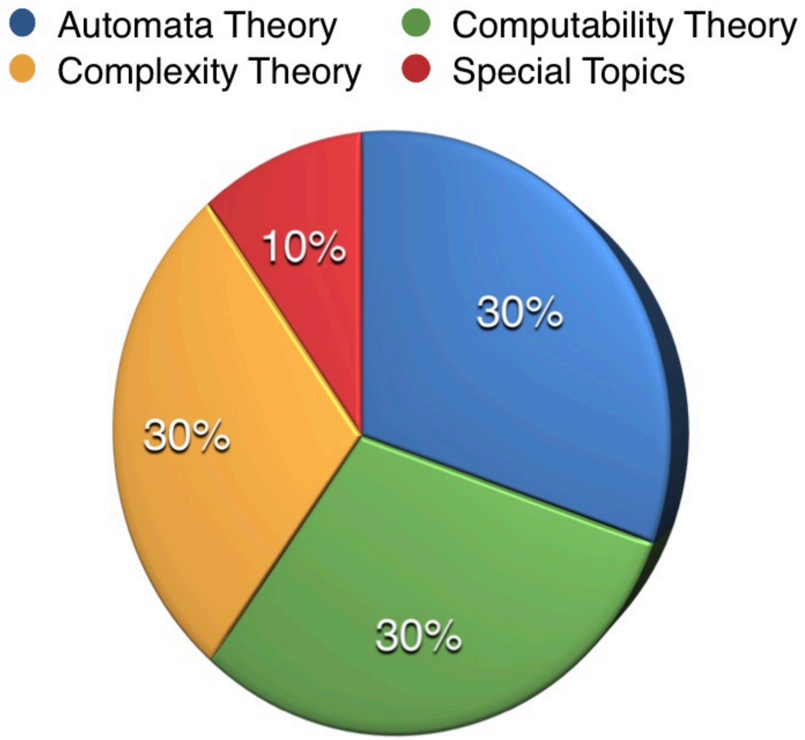
Theory of Computation

Automata Theory

Computability Theory

Complexity Theory

- **Complexity classes** : how to classify decidable problems based on their time and space requirements
- Complexity classes P and NP
- When a problem is called **intractable** (NP-completeness)
- Using reductions to prove problems intractable
- Space-complexity classes L and NL, PSPACE, and so on



Theory of Computation: (Rough) Schedule

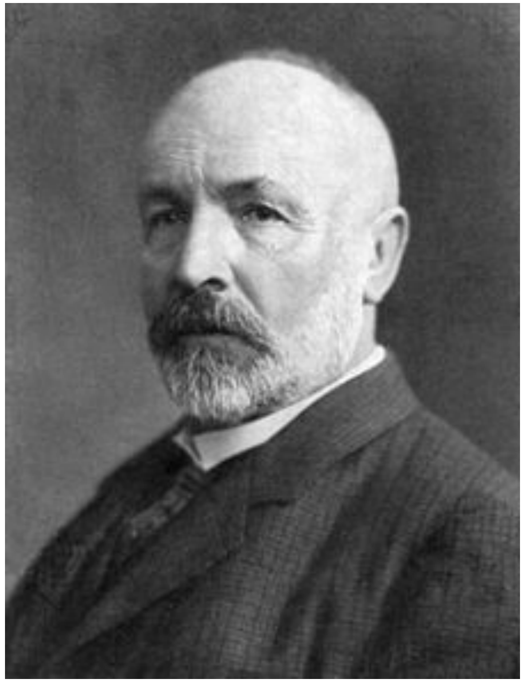
- Week 1 – Week 5 : Automata Theory (In-Class Quiz I)
- Week 6 – Week 10: Computability Theory (In-Class Quiz II)
- Week 11 – Week 15: Complexity Theory (In-Class Quiz III)
- Week 15 – Week 16: Special Topics

Special Topics

- Randomized Computation and Complexity
- Quantum Computation and Complexity
- Approximate Computation and Complexity
- Historical paper review
- Other? (suggestions welcome)

Discrete Mathematics: Review

Discrete Mathematics: Review



Georg Cantor
March 3, 1845 – January 6, 1918

- A set is a collection of objects, e.g.
  - $A = \{a, b, c, d\}$ and $B = \{b, d\}$
  - Empty set $\emptyset = \{\}$ (why it is not same as $\{\emptyset\}$)
  - $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ and $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
  - $\mathbb{Q}$ is the set of rational numbers.
  - $\mathbb{R}$ is the set of real numbers.
- $a \in A$: element of a set, belongs to, or contains
- Subset of $A \subseteq \mathbb{N}$, or proper subset of $A \subset \mathbb{N}$
- Notions of set union, intersection, difference, and disjoint
- Power set 2^A of a set A (example)
- Partition of a set

Discrete Mathematics: Review (Contd.)

- A ordered pair is a pair (a, b) of elements with natural order
- Similarly we define triplet, quadruplet, n-tuples, and so on
- Cartesian product $\mathrm{A}\times B$ of two sets is the set of orderd pairs

$$\mathrm{A}\times B = \{(a, b) : a \in A \; and \; b \in B\}$$

- Binary relation R on two sets A and B is a subset of $\mathrm{A}\times B$
- Recall definitions of
  - Reflexive, Symmetric, and Transitive relations,
  - and Equivalence relation.

Discrete Mathematics: Review (Contd.)

- A function (or mapping) f from set A to B is a binary relation s.t. sfor all $a \in A$ we have that $(a, b) \in f$ and $(a, b') \in f$ implies that $b = b'$.
- We often write $f(a)$ for the unique element b such that $(a, b) \in f$.
- Function $f: A \to B$ is one-to-one if for any two distinct elements $a, b \in A$ we have that $f(a) \neq f(b)$.
- Function $f: A \to B$ is onto if for every element $b \in B$ there is an element $a \in A$ such that $f(a) = b$.
- Function $f: A \to B$ is called bijection if it is both one-to-one and onto.

Cardinality of a Set

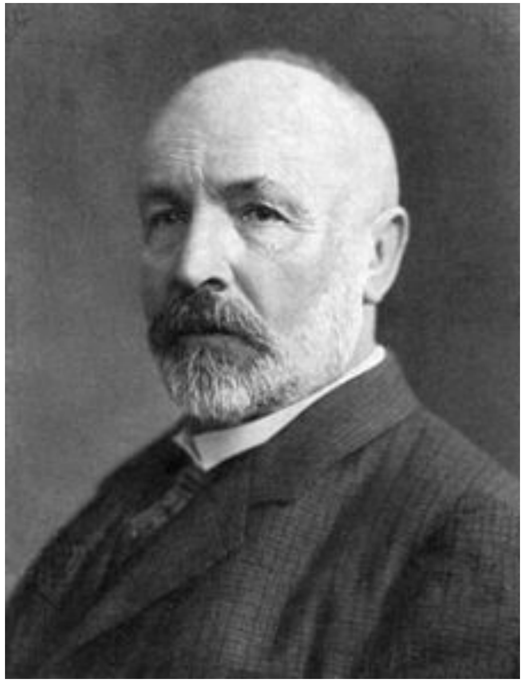
- Cardinality $|S|$ of a set S, e.g. $|A| = 4$ and $|\mathbb{N}|$ is an infinite number.
- Two sets have same cardinality if there is a bijection between them.
- A set is countably infinite (or denumerable) if it has same cardinality as $\mathbb{N}$.
- A set is countable if it is either finite or countably infinite.
- A transfinite number is a cardinality of some infinite set.

Theorem: Cardinality

Theorem

1. *The set of integers is countably infinite. (idea: interlacing)*
2. *The union of a finite number of countably infinite sets is countably infinite as well. (idea: dove-tailing)*
3. *The union of a countably infinite number of countably infinite sets is countably infinite.*
4. *The set of rational numbers is countably infinite.*
5. *The power set of the set of natural numbers has a greater cardinality than itself. (idea: contradiction, diagonalization)*

Cantor's Theorem

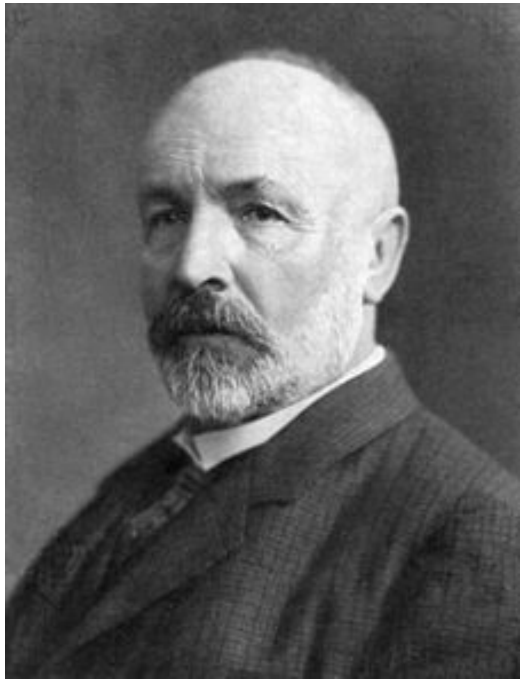


Theorem. *If a set* S *is of any infinite cardinality then its power set* 2^S *has a greater cardinality, i.e.* $|2^S| > |S|$.

(hint: happy, sad sets).

Corollary. *There is an infinite series of infinite cardinals.*

Cantor’s Theorem



Theorem. *There is an infinite series of infinite cardinals.*

“ a "grave disease" infecting the discipline of mathematics*”* –*Henri Poincaré*

“ "I don't know what predominates in Cantor's theory – philosophy or theology, but I am sure that there is no mathematics there"– Leopold Kronecker

“Most admirable flower of mathematical intellect” – David Hilbert